The Factors that Trigger the Transition to Turbulent Flow on the Keel of a Sailing Yacht

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Abstract. The right choice of the profile family for the keel of a sailing yacht depends on the location of the laminar to turbulent transition of the boundary layer. Possible causes for early transition and the turbulence level of the environment are discussed.

NOMENCLATURE

- \( B \): Beam of the ship
- \( c_p \): Phase velocity of a wave
- \( f \): Frequency
- \( g \): Gravitational acceleration = 9.81 m/s\(^2\)
- \( H_s \): Significant wave height
- \( k \): Wave number = \( 2\pi/\lambda \)
- \( r_g \): Sand grain roughness
- \( S \): Wave number spectrum
- \( \nu \): Kinematic viscosity of water
- \( \varepsilon \): Dissipation rate of turbulent kinetic energy
- \( \lambda \): Wavelength
- \( U \): Speed of the yacht
- \( U_f \): Speed of the yacht
- \( Tu \): Turbulence level
- \( TKE \): Turbulent kinetic energy
- \( u', v', w' \): Fluctuating velocities

1 INTRODUCTION

The choice of the profile for keel sections has always been a matter of debate. Usually naval architects specify profiles that were originally developed for airplanes [1]. Low drag symmetrical foil sections with extended natural laminar flow can be found in the NACA 6-series profile families [2]. Howlett [3] recommends a Wortmann section with a much wider laminar bucket, as it is used for gliders. Obara and van Dam [4] explain possible drawbacks of a design that has a very wide laminar bucket but come to the conclusion that the opportunities outweigh the risks. To make an educated decision on the choice of the section profile it is necessary to know the environment in which the foil will be used. If the water in the ocean induces a high turbulence level followed by early transition of the flow around the foil, a profile that is designed for extended natural laminar flow would be a bad choice. The NACA 4-digit profile family exhibits much better lift to drag ratios under these conditions. As a prerequisite, it is therefore necessary to investigate the possible causes of early transition.

2 TRANSITION TO TURBULENT FLOW AROUND THE KEEL

There are measurements at sea of the transition point of the keel-flow by an America's Cup team [5], but the results are kept secret, only tools and methods are described. The paper by Obara and van Dam [4] illustrates the importance of the attention to detail. They base their preference of laminar sections on experimental results by Carmichael who achieved extremely high Reynolds numbers without transition and concluded, that the "ambient turbulence level in parts of the ocean is low". A closer look at Carmichael's paper reveals that he conducted his experiments in a depth of 240 meters, which is not a safe place for sailing yachts. A step-by-step examination of possible causes for early transition follows in the next paragraphs.

2.1 The pitching motion of the yacht

If the bow of the yacht pitches up and down on a circle around the yacht's center of gravity, the keel tip will move forward and backward. This movement is superimposed on the speed of the yacht and can be regarded as a fluctuating velocity that could trip the boundary layer. The most violent pitching occurs in short steep waves with an encounter frequency close to the natural frequency of the yacht. The linear stability theory that explains the transition of the laminar b.l. is based on the occurrence of Tollmien-Schlichting waves. The frequencies that cause instability were measured [6] and can be calculated for the earliest possible transition

\[
f = \frac{24 \cdot 10^{-6}}{\nu} \cdot U^2
\]

(1)

and the lowest frequency that causes transition

\[
f = \frac{1.8 \cdot 10^{-6}}{\nu} \cdot U^2
\]

(2)
Let a yacht sail at a speed of 6 knots, then earliest transition will occur at a frequency of 208 Hz but below 16 Hz the b.l. will be stable. Since such high pitching frequencies will not be reached, classical linear transition will not occur.

Support for the importance of the frequency spectrum is also provided by Tanguay et al. [5]. He tried to reproduce in the wind tunnel the same high turbulence level that he measured in the ocean, using a fine grid to increase the turbulence artificially. The flow in the wind tunnel and the flow at full size in the ocean were not comparable at all. The high frequency of the turbulence in the wind tunnel caused early transition of the flow, whereas the low frequency of the turbulence at full size, due to a moderate pitching movement of the yacht, did not affect the laminar flow.

If we assume a boat length of 10 m and a pitching movement at the bow of 1 m/s, the fluctuating level would be 10% at 1 m below the center of gravity. This is sufficient to cause nonlinear bypass transition during the deceleration phase of the foil section. This type of pulsating flow is comparable to the similar condition of the unsteady flow along a rowing shell. Measurements for this case are reported by Day et al. [7]. The frequency in his experiments was 0.54 Hz, which is close to realistic pitching frequencies. Day et al. observed turbulent flow during deceleration and re-laminarisation during acceleration. The steady state Reynolds number for all tests was 2.9·10^7 and the fluctuating velocity was +22% and -34%. The average resistance coefficient during the rowing cycle was determined twice, once with free transition and once with transition fixed at 5% length. The resistance with free transition was 5% less than in the case with the b.l. trip. This indicates that laminar flow is possible and that it persists long enough to reduce the average resistance.

In the real world environment of a sailing yacht beating against the waves, we have to take into account the orbital motion of the water particles. While the bow is lifted by the crest of a short wave, the center of the hull will still be in the wave trough, where the water particles are moving towards the crest. The acceleration of the keel foil in the bow-up phase is accompanied by an acceleration of the water around the foil. Therefore, the acceleration of the water relative to the foil will most likely be less than what we have previously assumed. In summary, the pitching motion of the yacht will not cause constant premature transition. Laminar flow is still possible most of the time.

### 2.2 The orbital motions of the water particles in waves

A yacht sailing at 45° against wind and waves will encounter waves at a speed that is a superposition of a component of boat speed and the phase velocity of the wave. The frequency of encounter is

\[
f = \frac{c_x + 0.7 \cdot U}{\lambda}
\]  

(3)

If we assume deep water and set the encounter frequency equal to the lowest frequency that just triggers transition (equation 2), we get the upper limit of the boat speed where forced transition is possible from

\[
\frac{1.8 \cdot 10^{-6}}{v} U^2 - \frac{0.7 \cdot U}{\lambda} - \sqrt{\frac{g}{2 \cdot \pi \cdot \lambda}} = 0
\]  

(4)

This equation yields the highest boat speeds for the shortest wavelength. A wavelength of 1.1 m will give an upper boat speed of 2 kts. For higher boat speeds at this wavelength no early transition will occur. Longer waves will reduce this transition threshold below the value of 2 kts. On the contrary, a wavelength shorter than 1.1 m will lead to such a steep decay of the orbital motion with depth, that the fluctuating velocity at the keel level will be negligible. In summary we can therefore conclude, that the orbital motion will not trigger the transition, instead it can be regarded as a pulsating flow.

In long waves the pulsation of the flow will be attenuated by the fact that in reality the yacht is not sailing at constant speed. Rather than the speed, the sail forces are constant and the drag of the yacht, sailing against the waves, will depend on its speed relative to the water particles. The yacht will accelerate in the wave trough and decelerate on the crest, reducing the surge of the flow relative to the yacht.
2.3 Surface roughness and contamination

According to an equation by Braslow et al. [8], a roughness of 0.2 mm will surely trip the boundary layer at a boat speed of 6 kts. It is therefore necessary to keep the surface of the keel clean and smooth. Also in the turbulent part of the flow the allowed roughness is limited, if the surface is supposed to be hydraulically smooth. The allowed roughness depends on the thickness of the laminar sub-layer and is calculated by the textbook equation:

\[
\frac{r_g}{\nu} = 100 \frac{v}{U}
\]

At a speed of 6 kts, the roughness has to be less than 0.04 mm. A cruising yacht will not be polished constantly to this requirement, but for competitive sailing this is absolutely necessary.

2.4 Crossflow instability

On foils with a sweep angle of more than 45°, the crossflow caused by the favorable pressure gradient can be so severe, that the laminar b.l. is tripped. Details can be found in [4]. For the designer it is best to avoid larger sweep angles.

2.5 The angle of attack

The extended laminar flow on the laminar airfoils, which is indicated by the "laminar bucket" in the polar diagram, is caused by a favorable negative pressure gradient, which stabilizes the b.l. For increased angles of attack, the minimum of pressure on the suction side of the foil moves forward, very close to the leading edge. A separation bubble may occur and the flow will become turbulent. The limiting angle of attack for laminar flow is usually around 4 degrees.

2.6 The ambient turbulence level in breaking waves

The breaking of steep waves creates vorticity that could cause transition. Flick and George [9] have measured the turbulence length and velocity of breaking waves in the surf zone. They found out that the turbulent eddy length scales are on the order of the wave height and that the turbulence length scales and velocities decrease from the surface downward. Therefore, the situation in a plunging breaker is similar to the case of the orbital motion and the large-scale turbulence can be treated as a pulsating flow.

A different question is the development of the turbulence caused by wave braking over time. The kinetic energy that is contained in the breaking waves is cascaded down from large to small eddies in an inviscid process. Dissipation of the kinetic energy into heat occurs at the end of this cascade at the scale of molecular diffusion [10]. Therefore, a whole spectrum of eddy wavelengths from 1 mm to 1 m exists in the ocean surface layer. A sub-range of this spectrum are eddies with a turbulence scale of a few centimeters, which is the critical size that would cause early transition of the flow along the foil. This situation is discussed in the next paragraph.

2.7 The small scale ambient turbulence level

To measure the ambient turbulence level at the turbulence length scale that is small enough to trigger transition of the b.l. is not a simple task. The earliest indication was given by Kramer [11]. He started his tests in 1960 towing a streamlined cylinder of 1.2 m length at speeds between 2 and 20 knots and at a depth of 1 m below the surface. In an indirect process he inferred the transition Reynolds number from the measured drag and concluded that the fluctuating velocities remained almost constant at a given depth and for a given day. He deducted from his experiments an average fluctuating velocity of 0.01 m/s. The turbulence level that is defined as

\[
Tu = \frac{\sqrt{u'^2 + v'^2 + w'^2}}{\bar{U}}\frac{3}{3}
\]

for isotropic turbulence

\[
Tu = \sqrt{\frac{u'^2}{U}}
\]

varies therefore only with 1/U, i.e. fast ships "see" a lower turbulence level than slow ships. In this definition \(u'^2\) is not the total turbulent kinetic energy, but only that part of it that causes transition. In recent years, the turbulence in the ocean has been the subject of intense studies by oceanographers to understand the air-sea exchange processes and their impact on weather and climate. Gemmrich and Farmer [12] measured velocity profiles in the open ocean with acoustic Doppler sonars. At the small scales we are interested in, the turbulence...
can be assumed to be isotropic, e.g. the components $u'$, $v'$ and $w'$ have approximately the same values. In the inertial subrange the one-dimensional wave number spectrum can be represented by Kolmogorov's law, as shown in [12]:

$$S(k) = A \cdot \frac{1}{35} \cdot \varepsilon^{\frac{5}{3}} \cdot k^{\frac{5}{3}}$$  \hspace{1cm} (7)

$A = 1.5$ is a universal constant and the energy dissipation rate $\varepsilon$ is a slowly (compared to the frequency that causes transition) varying function of time and space. If we define an upper and lower limit of the wave number, e.g. the frequency band that can cause transition, we get the relevant fluctuating velocity by integration [10]:

$$\overline{u'^2} = \int_{k_1}^{k_2} S(k) \, dk$$  \hspace{1cm} (8)

The frequency band of interest is defined by equations 1 and 2. The eddy wavelengths (which should not be confused with the wavelength of the gravity waves in chapter 2.2) are:

$$\lambda_1 = \frac{2\pi}{k_1} = \frac{1.08}{U} \quad \text{U in (kts)} \quad \lambda \text{ in (m)}$$  \hspace{1cm} (9)

$$\lambda_2 = \frac{2\pi}{k_2} = \frac{0.08}{U} \quad \text{U in (kts)} \quad \lambda \text{ in (m)}$$  \hspace{1cm} (10)

Equation 10 would also give the appropriate value if a RANS simulation calls for the input of the turbulence length scale. The integration of equation 8 in the limits of equation 9 and 10 yields the fluctuating velocity as a function of the dissipation rate $\varepsilon$ of the turbulent kinetic energy (TKE). The resulting turbulence level is depicted in figure 1.

![Figure 1. Dependence of the turbulence level on the turbulent energy dissipation rate](image)

The value of the dissipation rate depends on the sea state. The following table gives an overview based on actual observations:

<table>
<thead>
<tr>
<th>region</th>
<th>inland lake</th>
<th>inshore waters</th>
<th>open sea, wave trough</th>
<th>breaking wave crest</th>
</tr>
</thead>
<tbody>
<tr>
<td>wind speed (kts)</td>
<td>10</td>
<td>14</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>sign. wave height (m)</td>
<td>&lt; 0.2</td>
<td>0.3</td>
<td>2.8</td>
<td>2.3-3.5</td>
</tr>
<tr>
<td>depth below surface (m)</td>
<td>1</td>
<td>1</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>$\varepsilon$ (W/kg)</td>
<td>$6.8 \cdot 10^{-7}$</td>
<td>$2.3 \cdot 10^{-6}$</td>
<td>$1 \cdot 10^{-4}$</td>
<td>$5 \cdot 10^{-7}$</td>
</tr>
<tr>
<td>reference</td>
<td>[13]</td>
<td>[14]</td>
<td>[12]</td>
<td>[12]</td>
</tr>
</tbody>
</table>
Figure 1 shows also Kramer’s experimental value. His results are highly plausible for tests in the open sea at one meter beneath the surface, considering the indirect method and the simple sensors that he used in 1960.

The NACA polar diagrams in [2] were determined in the Langley Two Dimensional Low Turbulence Pressure Tunnel, which has a turbulence level of 0.03%. This $Tu$-value is marked in figure 1. According to [6] a turbulence level of up to 0.1% will not affect the transition point, but above this level, the transition point will move forward on the foil and the laminar bucket will disappear. Based on this statement extended natural laminar flow will not be possible if the TKE dissipation rate is higher than $10^{-6}$ W/kg. As shown in the table, such low TKE values can be met in protected waters when there are almost no waves and only a light breeze is blowing. In this case, the TKE is transferred to the water only by the wind shear on the water surface. On the contrary, if wave breaking occurs, the TKE is drastically enhanced and its value exceeds the level predicted by wall layer theory by at least an order of magnitude. Observed values are shown in the table in the columns on the right. Laminar flow will not be possible under these circumstances. Terray et al. [15] propose a scaling for the rate of energy dissipation in the transfer layer below the wave-breaking layer, based on experimental findings. The transfer layer extends from a depth of $0.6 H_s$ below the still water level downward. If we assume the position of the keel at 1 meter below the wave trough, Terray’s scaling is applicable. Following his paper, it is possible to calculate $\varepsilon$ as a function of wind speed and wave height for a sea state with wave breaking. The peak spectral wave period that is needed to calculate phase speed was calculated from the JONSWAP spectrum [16]. The results are depicted in figure 2. It is obvious that in a sea with breaking waves above a wind speed of 14 kts the TKE dissipation rate is high enough to cause early transition of the flow along the keel.

![Figure 2. Turbulent kinetic energy dissipation rate at 1 meter below the wave trough for a sea state with breaking waves](image-url)

$2.8$ The bow wave

In the previous paragraph it was shown that breaking waves cause an increase of turbulent fluctuations in the water. It seems reasonable to suspect that a breaking bow wave could also increase the turbulence level around the ship. Roth et al. [17] measured the velocity fluctuations in the bow wave at a Froude number of 0.3 and at a Reynolds number of $1.6 \times 10^7$, both based on ship length. The resulting turbulence level in the definition of equation 6 was 10%. This high level is limited to the "toe" of the wave, where the spilling occurs. The turbulent production is concentrated in a thin layer against the forward face of the wave. The resolution in the diagrams in [17] is not fine enough to determine the turbulence level inside the wave. Figure 3 shows a cut through the bow wave from the hull to the toe using the data from [17]. Most likely, the turbulence in the middle of the wave is less than 1%. The orbital motion of the water particles will distribute this turbulence in a surface layer, which has the thickness of the wave height. It is not known, how the TKE is transferred and dissipated further downwards. Compared to the energy input of wind driven waves, which results in a downward energy flux on the entire
water surface, the small 3-D bow wave of limited extent will inject an energy flux that is confined to only a small strip along the hull. Looking at the steep decay of the turbulence away from the surface in figure 3, it is unlikely that significant portions of the TKE will reach the keel. It can be assumed, that the transition of the keel flow will not be influenced. This assumption will not hold true for the flow around the hull. The high turbulence level close to the bow will most likely cause early transition of the flow past the forward part of the hull.

2.9 Internal waves

Kramer reported in another paper [18], that the drag of his streamlined cylinder varied to his surprise with the season. The tests took place in Long Beach Harbor and near Catalina Island, California. He towed the cylinder at a depth of 1 m with a speed of 26 kts. The measured drag increased dramatically between April and August, having a peak in July. The seasonal change of the drag followed the radiation heat flux from the sky. If the density of the water increases from the surface downward because the temperature decreases, the surface layer is stably stratified. During the colder seasons the upper part of the ocean is well mixed and the temperature in the surface layer is constant. The drag increase caused by density stratification was described in the scientific literature for the first time in 1897 by Fridtjof Nansen, who experienced such a situation with his ship Fram. He named the phenomenon "dead water". The drag increase is an additional wave resistance caused by the formation of internal gravity waves. Instructive experiments, including the production of a video, were lately conducted by Mercier et al. [19]. Additional random gravity waves originate in the turbulent wake of the body and increase the turbulent energy inside the fluid behind the body [20]. Applied to the sailing yacht, the turbulence in the internal gravity waves created below the bow could influence the flow around keel and rudder negatively.

3 THE CHOICE OF THE RIGHT PROFILE FAMILY

Summarizing the results of the previous paragraph, extended natural laminar flow with the benefit of low viscous drag is possible if:

- the yacht is often reaching or running with a drift angle of less than 4 degrees
- the yacht will mostly sail on lakes or inshore waters without wave breaking
- the surface of the keel will be kept smooth and polished all the time
- the keel is straight without a large sweep angle

Since these circumstances will not prevail all of the time, the profile should also work well in a more turbulent environment. The NACA 63-012 wing section would be a good compromise that enables natural laminar flow, but also has a good lift to drag ratio in turbulent waters. If the owner wants a yacht solely optimized for blue water sailing, the NACA 0012 section should be considered, as it gives lower drag values at all drift angles for turbulence levels above 0.1%. Since this profile was developed 83 years ago, it would be interesting to know, if better profiles for turbulence levels around 0.5% could be developed with today's CFD-tools. This might be a chance for performance increases that has not yet been pursued.
4  ACKNOWLEDGEMENTS
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5  REFERENCES