

# Resistance Prediction for Sailing Yacht Hulls Based on Systematic Towing Tank Tests

Copyright © Ulrich Remmlinger, Germany, 2017

**Abstract.** A regression analysis of the bare hull upright resistance of the DSYHS was presented in a previous paper. This new regression includes also tank test results of heeled and appended hulls of the DSYHS and in addition those of the US Sailing Nine Model Series.

## NOMENCLATURE

$A_{Stm}$	Stem angle, positive forward	$L_{TO}$	Length of transom overhang
$A_{W1}$	Water plane area, forward of max. section	$L_{WL}$	Length of the actual water line at rest with trimming moment applied
$A_{W2}$	Water plane area, aft of max. section		
$A_X$	Area of max. section = maximum immersed area of all cross sections	$L_{WLO}$	Length of the designed water line
$ATK$	Angle of attack, see text	$R_{res}$	Residuary resistance force
$BM$	Longitudinal metacentric height	$ROK$	Exit angle in the heeled lateral plane
$B_X$	Beam in the heeled water plane at max. section	$T_X$	Draft of heeled canoe body at max. section
$c_{keel}$	Chord length of keel at root	$t_{keel}$	Thickness of keel at root
$Fn$	Froude number $U_\infty / (g \cdot L_{WL})^{1/2}$	$U$	Water speed at edge of boundary layer
$g$	gravitational acceleration = 9.81 m/s <sup>2</sup>	$U_\infty$	Ship speed
$I_E$	Incidence angle at water plane entrance	$V_{atm}$	Volume attenuated with depth, see [1]
$E_{wp}$	Exit angle at stern of elevated waterplane	$V_{CB}$	Displaced volume of canoe body
$L_1$	Distance from front of $L_{WL}$ to max. section	$V_1$	Part of $V_{CB}$ forward of max. section
$L_2$	Distance from max. section to aft end $L_{WL}$	$V_2$	Part of $V_{CB}$ aft of max. section
$Lcb$	Distance front of $L_{WL}$ to centre of buoyancy	$x, y, z$	Coordinate system, $x$ -direction from stern to bow, $z$ -direction upwards
$Lcf$	Distance front of $L_{WL}$ to centre of flotation	$\rho$	Density of the water
		$\sigma$	Standard deviation

## 1. INTRODUCTION

A regression analysis of the bare hull upright resistance of the Delft Systematic Yacht Hull Series (DSYHS) was presented in a previous paper [1]. For sailing yachts it is desirable to extend this analysis and include also all tank tests with appended hulls and also those that were performed in a heeled attitude. Since the publication of the first paper [1] additional tank test results became available. In 2015 the Sailing Yacht Research Foundation (SYRF) published results of a tank test program from 2003 that was until then not available in the open literature. The tested fleet is called the US Sailing Nine Model Series [2]. This was a great opportunity to extend the database for the regression analysis.

In the original Delft-method the effect of heel is treated as an additional resistance that is added to the value of the upright resistance [3]. The method that was introduced in [1] allows the determination of the resistance at any given attitude, heeled or upright, without calculating the upright resistance as an intermediate step. With the addition of the heeled and appended models of the DSYHS and also all the USSAIL-models, the total number of interpolated towing tank results at fixed Froude-numbers increased from 1276 to 2718 experimental data points. The data points are spaced at  $Fn$ -intervals of 0.05 between  $Fn = 0.1$  and 0.8. In the medium  $Fn$ -range each regression at a fixed  $Fn$  uses the data of approx. 250 independent test-runs ( $N$ ) of different hull forms or hull attitudes. The exact numbers for  $N$  are given in table 1. No other regression analysis in the open literature is based on such a large number of different tests.

## 2. CORRECTION OF THE RAW DATA

According to an E-mail exchange with the researchers at Delft, the measured resistance values were only corrected for the parasitic drag of the sand strips. No corrections were applied for the blockage effects in the tank. The results published by the SYRF are raw data, without any correction, except for mechanical crosstalk. The aim of a resistance prediction is the prediction of the drag force in unrestricted waters of infinite depth. Therefore the measured drag values had to be corrected for the blockage effect and the finite depth in the towing tank. At Delft the experiments were conducted in the No.1 towing tank, which has a width of 4.22m and a depth of 2.50m [4]. The towing tank of the IMD at St. John's is 12m wide and 7m deep [5]. Schuster's method [6] and Tamura's model of a Rankine Ovoid [7] were used to correct the measured speed to the equivalent speed at

infinite depth. The measured force was corrected according to Tamura [7], which is especially necessary at the high Froude-numbers in the Delft-tank with its limited depth. The corrections amounted up to 10%.

### 3. VISCOUS RESISTANCE OF THE HULL WITH APPENDAGES

The viscous resistance is computed for the heeled hull in the same way as in the upright case [1]. The planforms and profiles of keel and rudder were taken from the CAD-models that came with the test results. The DSYHS uses the profile NACA 63<sub>2</sub>A015 for the keel and NACA 0012 for the rudder. USSAIL-models have profiles NACA 64<sub>2</sub>A013 for both keel and rudder. The viscous resistance of the appendages was determined by an integration of the profile drag at 8 different sections along the span of the fin. The section drag was computed with the program XFOIL [8] as a function of the Reynolds-number. Polynomials of higher order were fitted to the drag curves, so as to enable interpolation of the profile drag for each desired Reynolds-number. The total viscous resistance of the yacht was assumed to be the sum of the viscous resistance of hull, keel and rudder, plus an interference resistance as described in [9]. The residuary resistance  $R_{res}$  is calculated by subtracting the total viscous resistance from the measured resistance.

### 4. SELECTION OF THE PARAMETERS

The aim of the regression analysis is the determination of appropriate coefficients for the prediction of the residuary resistance of the hull [1]. All textbooks on statistical methods stress the importance of avoiding collinearity, when selecting the dimensionless variables for the regression analysis. Therefore in the initial paper [1] a large portion was devoted to the inspection of the database and to the analysis of the problem of collinearity. The variables that were used were mostly composed of global dimensions, like total length, beam, volume etc. The dimensional analysis gave the smallest number of dimensionless variables that contained all of those global dimensions. The variables were chosen in such a way that collinearity was reduced. When the database was now increased with additional hull forms [2] it soon became evident, that the original small set of variables with only the global dimensions does not suffice to describe the resistance of a wide variety of hull forms. It seems that the resistance does not only depend on global dimensions but is also heavily influenced by local changes in hull properties like e.g. the local curvature of the hull. More than 100 new parameters were tested in the regression analysis with the hope to find variables that improve the accuracy of the predicted hull resistance. Several local angles were defined such as the flare angles at different stations and entrance and exit angles of different waterplanes. The residual mean square [10] was used to identify good parameter combinations. In the end it turned out that variables that are almost collinear are still valuable because the subtle differences between nearly collinear variables of global parameters seem to contain valuable information about local hull properties. The new strategy for finding a helpful regression did therefore not look at a possible collinearity any more, instead the residual mean square and the absolute sum of the coefficients were minimized. Under this strategy a set of 35 dimension-less variables was defined for the canoe body and two additional variables for the keel:

$$\begin{aligned}
 BL_1 &= \frac{B_X}{L_1} & BL_2 &= \frac{B_X}{L_2} & TL_1 &= \frac{T_X}{L_1} & TL_2 &= \frac{T_X}{L_2} & AW_1 &= \frac{\sqrt{A_{W1}}}{L_1} & AW_2 &= \frac{\sqrt{A_{W2}}}{L_2} & AX_1 &= \frac{\sqrt{A_X}}{L_1} & AX_2 &= \frac{\sqrt{A_X}}{L_2} \\
 LV_1 &= \frac{L_1}{\sqrt[3]{V_1}} & LV_2 &= \frac{L_2}{\sqrt[3]{V_2}} & VA_1 &= \frac{\sqrt[2]{V_1}}{A_{W1}} & VA_2 &= \frac{\sqrt[2]{V_2}}{A_{W2}} & C_{P1} &= \frac{V_1}{A_X \cdot L_1} & C_{P2} &= \frac{V_2}{A_X \cdot L_2} & C_{V1} &= \frac{V_1}{A_{W1} \cdot T_X} & C_{V2} &= \frac{V_2}{A_{W2} \cdot T_X} \\
 C_{W1} &= \frac{A_{W1}}{L_1 \cdot B_X} & C_{W2} &= \frac{A_{W2}}{L_2 \cdot B_X} & C_X &= \frac{A_X}{B_X \cdot T_X} & U_{04} &= \frac{U_{4\%}^2}{U_\infty^2} & U_{25} &= \frac{U_{25\%}^2}{U_\infty^2} & U_{\max} &= \frac{U_{\max}^2}{U_\infty^2} & I_E & ROK & A_{ST} \\
 LCB &= \frac{Lcb}{L_{WL}} & LCF &= \frac{Lcf}{L_{WL}} & Fl_B &= \left. \frac{dy}{dz} \right|_{Bow} & Fl_S &= \left. \frac{dy}{dz} \right|_{Stern} & Fl_X &= \left. \frac{dy}{dz} \right|_{\max. section} & BML &= \frac{BM}{L_{WL}} & BT &= \frac{B_X}{T_X} & E_{WP} \\
 OVH &= e^{-20 \cdot L_{TO} / L_{WL}} & ATK & t_K B &= \frac{t_{keel}}{B_X} & c_K L &= \frac{c_{keel}}{L_{WL}}
 \end{aligned}$$

A few remarks are necessary to clarify the definition of some variables.  $U_{4\%}$  is the water speed at the edge of the boundary layer 4% of  $L_{WL}$  behind the forward end of the  $L_{WL}$ ,  $U_{25\%}$  at 25% of  $L_{WL}$ .  $U_{\max}$  is the maximum water speed along the hull.  $ROK$  is the angle (rocker) between the lower line of the lateral plane and the water plane at the aft end of the  $L_{WL}$ .  $Fl$  is the flare angle.  $E_{WP}$ , the exit angle of a waterplane, is the difference in half beam between max. section and stern at an elevated waterplane  $T_X/3$  above the designed water plane, divided by  $L_2$ . The reason that all angles are calculated from the half-beam divided by the x-distance is because the waterplanes of the heeled hull are not symmetric to the centerline.  $L_{TO}$  (transom overhang) is the distance between the aft end

of the  $L_{WL}$  and the aft end of the hull.  $ATK$  is an angle of attack that is different from zero only in the heeled attitude. If the hull heels, the line through the half beam points is not straight but curved. The distance between this curve and the straight line from bow to stern at the position of the max. section is taken and divided by  $L_l$  to yield  $ATK$ .

The denominator of the dependent variable  $Y_{CB}$  for the resistance of the canoe body is defined in three different ways. The different versions will be used for different  $Fn$ -ranges, as listed at the end of table 1.

$$Y_{CBI} = \frac{R_{res,CB}}{\frac{1}{2} \rho \cdot U_{\infty}^2 \cdot \frac{V_{atm,CB}}{L_{WL}}} \quad Y_{CBII} = \frac{R_{res,CB}}{\frac{1}{2} \rho \cdot U_{\infty}^2 \cdot (A_{W1} + A_{W2})} \quad Y_{CBIII} = \frac{R_{res,CB}}{\frac{1}{2} \rho \cdot U_{\infty}^2 \cdot \frac{V_{CB}}{L_{WL}}}$$

The residuary resistance of the keel is made dimensionless according to:

$$Y_{keel} = \frac{R_{res,keel}}{\frac{1}{2} \rho \cdot U_{\infty}^2 \cdot \frac{V_{atm,keel}}{c_{keel}}}$$

The total residuary resistance  $R_{res}$  is assumed to be composed of  $R_{res,CB} + R_{res,keel}$ . At each fixed  $Fn$  the method of full search was employed for the choice of the best subset out of the 37 variables.

## 5. RESULTS OF THE REGRESSION ANALYSIS

### 5.1 Linear regression

For the linear regression a number of 16 variables for the resistance of the canoe body and 2 variables for the keel gave the best results. Using the nomenclature that was introduced in [1] one can write for the  $Fn$ -range of  $Y_{CBI}$ :

$$Y_i(\mathbf{X}_i) = \beta_0 + \sum_{j=1}^{16} \beta_j X_{i,j} + \frac{V_{atm,keel} / c_{keel}}{V_{atm,CB} / L_{WL}} \left( \beta_{17} + \sum_{j=18}^{19} \beta_j X_{i,j} \right) + \varepsilon_i$$

The denominator of  $Y$  is the same as for  $Y_{CB}$ . For  $Y_{CBII}$  and  $Y_{CBIII}$  the denominator in front of the bracket changes accordingly. Including the two intercepts  $\beta_0$  and  $\beta_{17}$  there are 20 coefficients that need to be determined. The variables 1 to 16 for the canoe body have to be chosen as a subset out of the 35 available variables, the variables 17 to 19 for the keel are always included for  $Fn \geq 0.3$ . The first condition for the choice of the best subset is a very small residual sum of squares  $RSS$ :

$$RSS = \sum_{i=1}^N \varepsilon_i^2$$

$N$  is the sample size, which is listed in table 1 for each fixed  $Fn$ . The second condition is a small absolute sum of the coefficients. For this criteria all variables have to be centered and standardized [10], i.e.  $X$  and  $Y$  have zero means and standard deviations equal to one. The absolute sum should fulfill:

$$\sum_{j=1}^{19} |\beta_j| \leq s$$

Looking at the possible subsets that are found in the full search there is always a tradeoff between a low  $RSS$  and a low sum of  $|\beta|$ . The number 10 is a good upper threshold  $s$  in this linear regression with 20 coefficients. In case of standardized variables the coefficients  $\beta$  describe the influence of the variable  $X$  on the prediction  $Y$ . If there are large coefficients with alternating signs and the absolute sum of  $\beta$  is too large, the prediction of the resistance

will become very sensitive to small changes in the variables and the predicted values might play havoc if the hull form lies slightly outside of the database.

The chosen variables and the coefficients for the standardized variables are listed in table 1. Because all variables are centered, all  $\beta_0$  are equal to zero. The choice of the variables is of course arbitrary. A different set of variables could give similar results. The physically "right" parameters are disguised in the "noise" created by the measurement errors.

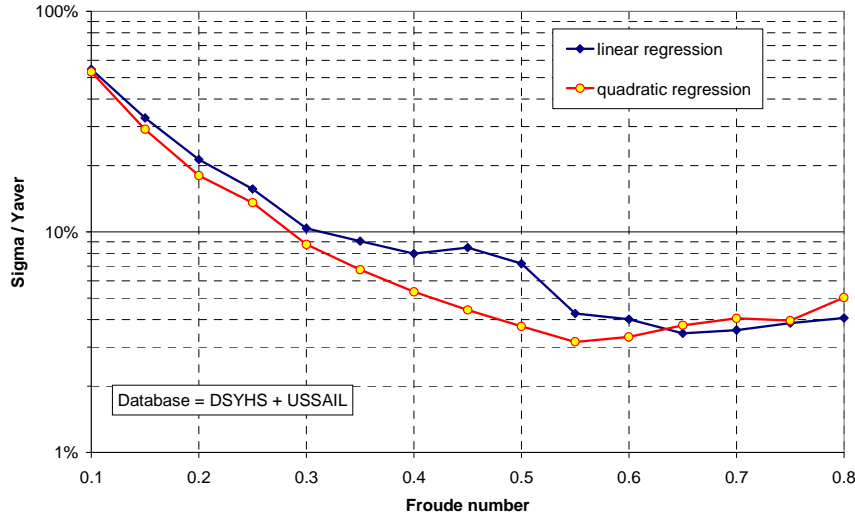
$F_n$	$N$	fore body													mid ships					
		$C_{P1}$	$BL_1$	$TL_1$	$AW_1$	$LV_1$	$VA_1$	$C_{W1}$	$C_{V1}$	$AX_1$	$I_E$	$U_{04}$	$U_{25}$	$Fl_B$	$A_{Sm}$	$C_X$	$Fl_X$	$BT$	$U_{max}$	$ATK$
0.10	151				-0.20	-0.14				-0.21		0.52	0.03		0.50		0.07	-0.40		0.15
0.15	180				-3.81	-1.63				2.32		0.45	0.01		0.37		-0.28	1.14		0.29
0.20	215				-2.39	-0.67				1.10		0.34	0.03		0.42		-0.25	0.94		0.27
0.25	245				-1.37	-0.20				0.26		0.28	0.07		0.34		-0.20	0.84		0.23
0.30	250	0.15		-1.31					0.29				0.71	0.14		1.46				0.19
0.35	254	0.24	0.50	-1.04							-0.06		0.24				-0.13		0.76	0.11
0.40	254	0.13		0.76	0.46		0.59		1.19				-0.27						0.71	0.05
0.45	254	0.17	1.54							-1.04			-0.43	-0.40			-0.31	0.37	0.77	0.19
0.50	216		0.94						1.20				-0.32	-0.30				0.73	0.26	0.11
0.55	186		0.48	-0.37					0.93	0.78	-0.06		-0.12	-0.15						0.06
0.60	164		0.49	-0.49					1.16	1.02	-0.06		-0.10	-0.16						0.07
0.65	116		0.20					-0.07	0.71	1.10	-0.20			-0.15					-0.51	0.07
0.70	92		0.27					-0.05	1.12	1.29	-0.20			-0.14					-0.65	0.05
0.75	79		1.22		-1.11			-0.31			-0.23	-0.23	0.14	-0.08				-1.27		0.06
0.80	62		1.32		-1.20			-0.29			-0.21	-0.17	0.24	-0.06				-1.44		0.05

$F_n$	entire hull			after body													keel			
	$LCF$	$LCB$	$BML$	$C_{P2}$	$BL_2$	$TL_2$	$AW_2$	$LV_2$	$VA_2$	$C_{W2}$	$C_{V2}$	$AX_2$	$ROK$	$Fl_S$	$E_{WP}$	$OVH$	$\beta_{17}$	$t_{KB}$	$c_{RL}$	$Y_{CB}$
0.10	1.01	-0.81		-0.40	0.37				-0.72		0.16		0.80							$Y_{CB1}$
0.15	-1.09	-0.57		0.46	1.07				-2.08		-0.25		0.12							$Y_{CB1}$
0.20	-0.30	-0.84		0.22	0.88				-1.39		-0.24		0.67							$Y_{CB1}$
0.25	0.08	-0.86		0.14	0.62				-0.82		-0.21		0.92							$Y_{CB1}$
0.30			-0.26	0.78		0.88				-1.42	-0.99		0.72	0.22	-0.16	0.09	-0.35	-0.50	0.97	$Y_{CBII}$
0.35			0.41				-0.61	-0.76		-0.28	-0.09		0.76		-0.44	0.04	-0.21	-0.25	0.63	$Y_{CBII}$
0.40	-1.58	1.31							-0.43		-0.45	-0.89	0.18	-0.01		0.00	-0.05	-0.14	0.33	$Y_{CBIII}$
0.45	-0.61	0.43			-1.12			-0.95			-0.39		0.34			0.03	-0.42	-0.43	1.13	$Y_{CBIII}$
0.50	-2.02	1.34			-0.32	0.45				0.50	-0.59		0.18	-0.70		0.04	-0.11	-0.03	0.31	$Y_{CBIII}$
0.55	-1.22	0.92			0.07		0.55		0.35		-0.39		0.29			0.08	-0.11	0.04	0.22	$Y_{CBIII}$
0.60	-1.43	1.12			0.11		0.63		0.48		-0.47		0.18			0.07	-0.13	0.00	0.29	$Y_{CBIII}$
0.65	-0.94	0.78						0.55			-0.32	0.93	0.22	0.19		0.07	0.03	0.15		$Y_{CBIII}$
0.70	-1.24	1.23						0.71			-0.50	0.97	0.22	0.14		0.08	0.13	0.09		$Y_{CBIII}$
0.75	-0.82		0.47	0.27					-0.38			1.17		1.10		0.11	0.32	-0.05		$Y_{CBIII}$
0.80	-0.73		0.50	0.23					-0.35			1.00		1.24		0.07	0.35	-0.08		$Y_{CBIII}$

Table 1. Coefficients of the selected variables in the linear full search

The relative error for the prediction of the resistance  $R_{res}$  is depicted in figure 1. The standard deviation  $\sigma$  is divided by  $\bar{Y}$ , the average of  $Y_i$  for  $i = 1$  to  $N$ . The denominator under the root sign represents the degrees of freedom of the error vector. At low Froude-numbers the residuary resistance  $\bar{Y}$  is very small, therefore the relative error increases as  $F_n$  decreases.

$$\frac{\sigma}{\bar{Y}} = \sqrt{\frac{RSS}{N-20}}$$



**Figure 1. Relative standard deviation for the prediction of  $R_{res}$**

For some readers it might be surprising that the selection of variables differs significantly from that one of the upright case as described in [1]. This can be explained by the collinearity of the variables. The inclusion of the heeled and appended test runs requires additional variables and increases the measurement noise. In such a case the variable selection by full search is driven to a certain degree by random errors.

## 5.2 Quadratic regression

For the quadratic regression 8 variables for the resistance of the canoe body and 2 variables for the keel gave the best results. Using the nomenclature that was introduced in [1] one can write for the  $F_n$ -range of  $Y_{CBI}$ :

$$Y_i(\mathbf{X}_i) = \beta_0 + \sum_{j=1}^{44} \beta_j X_{i,j} + \frac{V_{atn,keel} / C_{keel}}{V_{atn,CB} / L_{WL}} \left( \beta_{45} + \sum_{j=46}^{47} \beta_j X_{i,j} \right) + \varepsilon_i$$

The 44  $\beta_j X_{ij}$  for the canoe body are made up from 8 linear, 8 quadratic and 28 mixed terms. Together with the two intercepts and the two keel variables a sum of 48 coefficients have to be determined in the full search. Again the choice is guided by a low  $RSS$  and a low sum of  $|\beta|$ . The absolute sum of the coefficients varied from 5 to 15. The chosen variables are listed in table 2,  $Y_{CB}$  differs from the linear case. The relative error for the prediction of the resistance  $R_{res}$  is depicted in figure 1. From this figure it is obvious that the quadratic regression is only beneficial in the medium  $F_n$ -range. At low and high Froude-numbers the residuary resistance can be described sufficiently by linear functions.

$F_n$	$N$	fore body													mid ships					
		$C_{Pl}$	$BL_l$	$TL_l$	$AW_l$	$LV_l$	$VA_l$	$C_{wl}$	$C_{vl}$	$AX_l$	$I_E$	$U_{04}$	$U_{25}$	$Fl_B$	$A_{Sm}$	$C_X$	$Fl_X$	$BT$	$U_{max}$	$ATK$
0.10	151	X			X								X		X		X	X		X
0.15	180	X	X											X	X			X		X
0.20	215	X												X	X	X		X		X
0.25	245	X												X	X	X		X		X
0.30	250												X							X
0.35	254	X				X			X									X		X
0.40	254						X			X									X	X
0.45	254		X				X			X		X							X	X
0.50	216								X	X		X						X		X
0.55	186																			X
0.60	164																			X
0.65	116			X						X										X
0.70	92			X			X													X
0.75	79				X		X	X		X										X
0.80	62				X			X		X										X

<i>F<sub>n</sub></i>	<i>entire hull</i>			<i>after body</i>													<i>keel</i>			<i>Y<sub>CB</sub></i>	
	<i>LCF</i>	<i>LCB</i>	<i>BML</i>	<i>C<sub>P2</sub></i>	<i>BL<sub>2</sub></i>	<i>TL<sub>2</sub></i>	<i>AW<sub>2</sub></i>	<i>LV<sub>2</sub></i>	<i>VA<sub>2</sub></i>	<i>C<sub>W2</sub></i>	<i>C<sub>V2</sub></i>	<i>AX<sub>2</sub></i>	<i>ROK</i>	<i>Fl<sub>S</sub></i>	<i>E<sub>WP</sub></i>	<i>OVH</i>	<i>β<sub>45</sub></i>	<i>t<sub>kB</sub></i>	<i>c<sub>R</sub>L</i>		
0.10				X																	<i>Y<sub>CB1</sub></i>
0.15						X				X											<i>Y<sub>CB1</sub></i>
0.20		X								X											<i>Y<sub>CB1</sub></i>
0.25		X								X											<i>Y<sub>CB1</sub></i>
0.30		X				X						X	X		X	X	X	X	X	X	<i>Y<sub>CBII</sub></i>
0.35				X		X										X	X	X	X	X	<i>Y<sub>CB1</sub></i>
0.40		X					X				X					X	X	X	X	X	<i>Y<sub>CB1</sub></i>
0.45									X							X	X	X	X	X	<i>Y<sub>CBIII</sub></i>
0.50			X										X			X	X	X	X	X	<i>Y<sub>CBIII</sub></i>
0.55																X	X	X	X	X	<i>Y<sub>CBIII</sub></i>
0.60																X	X	X	X	X	<i>Y<sub>CBIII</sub></i>
0.65		X					X									X	X	X	X	X	<i>Y<sub>CBIII</sub></i>
0.70		X			X						X					X	X	X	X	X	<i>Y<sub>CBIII</sub></i>
0.75				X		X										X	X	X	X	X	<i>Y<sub>CBIII</sub></i>
0.80				X				X								X	X	X	X	X	<i>Y<sub>CBIII</sub></i>

**Table 2. Selected variables in the quadratic full search**

Above  $F_n = 0.6$  the number of hull-variables was reduced from 8 to 7 because the smaller sample size  $N$  would cause an overprediction with the initial 48 variables.

## 6. PREDICTED VS. EXPERIMENTAL RESISTANCE

Figure 2 compares the predicted and experimental values of the total resistance for the new linear regression and for the Delft-method. The linear regression is used, because it is even applicable if the hull form of interest does not match exactly the hull forms of the database. The quadratic regression would give much better results for the database, but here we examine the less favorable case. The Delft-method uses 24 regression coefficients, including the effects of trim and heel.  $Y_{tot}$  in the diagrams is defined in the following equation.  $R_{tot}$  is either the measured total resistance or the sum of viscous and residual resistance.

$$Y_{tot} = \frac{R_{tot}}{\frac{1}{2} \rho \cdot U_{\infty}^2 \cdot \frac{V_{CB}}{L_{WLO}}}$$

In the ideal case all points would be lying on the diagonal line. A high value of the kurtosis is a clear sign that the distribution is peaked and not normal (Gaussian). The red dashed lines in figure 2 indicate the  $2 \cdot \sigma$  bands. In case of a normal distribution of the errors, the  $\pm 2 \cdot \sigma$  band would contain 95% of all test points. Because of the high kurtosis the distribution is not normal. With a sample size of 3518 it is possible to determine the quantiles empirically by counting. The results (94%) are in both cases almost identical to the values of the normal distribution. The statistic evaluation shows that in the new formula the standard deviation and also the error band are reduced by a factor of 2.2 compared to the Delft-method.

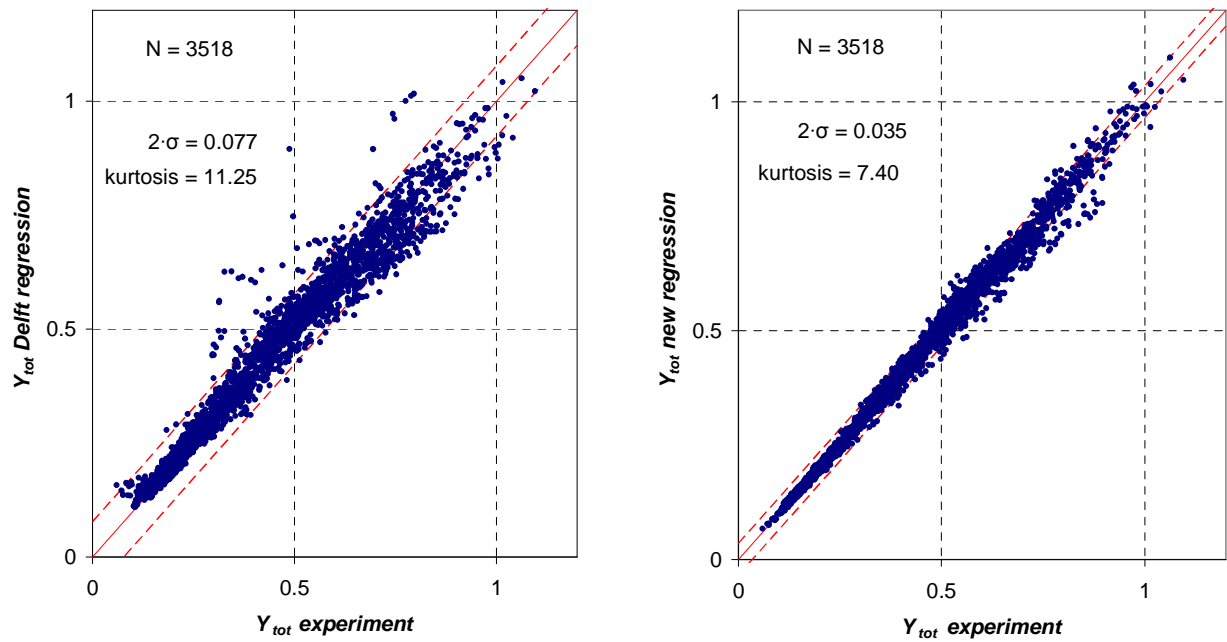


Figure 2. Total resistance coefficients for all experiments  $0.1 < Fn < 0.8$

The relative error of the predicted total resistance is depicted in figure 3. The prediction error is calculated from

$$Y_{error} = Y_{tot\ predicted} - Y_{tot\ measured}.$$

The  $\pm 2\cdot\sigma$  band is reduced with the new method from  $\pm 20.4\%$  to  $\pm 7.2\%$ . The relative error is obviously larger at the low end for small  $Fn$ . This trend is more pronounced in the Delft regression than in the new one. The reduction of the errors is significant. It must be pointed out, that these errors are only valid for models that are not too different from the tested models. For models far outside of this database the error will be larger because of the selection bias in the regression analysis. On the contrary, for the models that are part of the database the quadratic regression can be used. In this case the  $2\cdot\sigma$  band is only  $5.5\%$ .

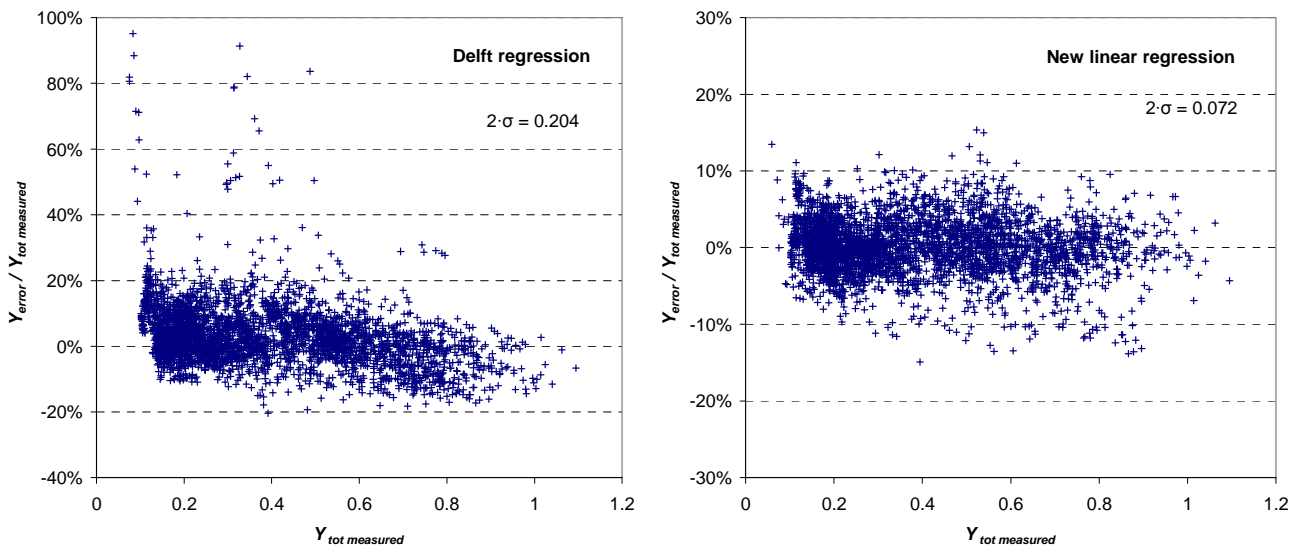
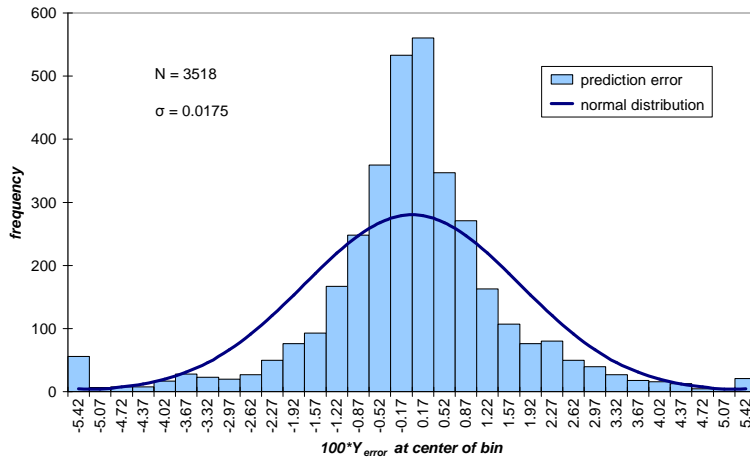
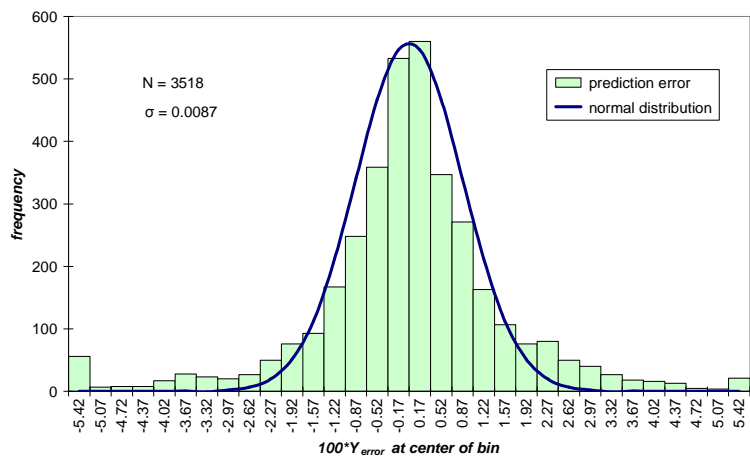


Figure 3. Total relative resistance error for all experiments  $0.1 < Fn < 0.8$

The kurtosis of the final selection is still high and the distribution is not normal. It is therefore interesting to have a look at the statistical distribution of the prediction errors for the new regression. The deviation from the normal distribution with a peak at the mean and missing values at medium distance to the mean is clearly visible in figure 4. Another way to look at this picture is the assumption of a normal distribution caused by the measurement errors and on top a superposition of a random error caused by an unknown parameter. Figure 5 depicts this hypothesis by assuming a smaller standard deviation for the normal distribution.



**Figure 4. Distribution of the prediction error and comparison with normal distribution of equal standard deviation**



**Figure 5. Distribution of the prediction error and comparison with a normal distribution of reduced standard deviation**

The unknown parameters that cause additional randomly distributed small and large errors are to a minor extent the quadratic terms that are left out in the linear model, but in addition it might be a hull parameter that was not yet considered or it could also be the influence of the towing tank set up. The roughness strips are changed between the test runs, this could have an influence. Normally the measured resistance is time dependent and an averaging process is employed, which is unknown. Without a detailed knowledge of the experimental process and ideally the inspection of the raw data, a further analysis is not possible and the current error must be accepted.

## 7. CONCLUSION

The new regression model improves the prediction of the resistance of the hull of a sailing yacht compared to the Delft-method. Future comparisons will tell, if the improvements will consistently appear also with different and new designs. To enable this necessary feed-back, the new prediction-software "UliTank" was developed and is available online [11].

## 8. REFERENCES

1. Remmlinger, U., "Bare Hull Upright Resistance Prediction Based on the Delft Systematic Yacht Hull Series", 2015, [Online]. Available: <http://www.remmlinger.com/Regression%20DSYHS.pdf>



2. Teeters, J., Pallard, R., Muselet, C., "US Sailing Nine Model Series", 2003, [Online]. Available: <http://sailyachtresearch.org/resources/us-sailing-nine-model-series>
3. Keuning, J.A., Sonnenberg, U.B., "Approximation of the Hydrodynamic Forces on a Sailing Yacht based on the DSYHS", *HISWA Symposium*, Amsterdam, NL, 1998
4. <https://www.tudelft.nl/en/3me/organisation/departments/maritime-and-transport-technology/research/default-title/facilities/towing-tank-no-1/>
5. [https://www.nrc-cnrc.gc.ca/eng/solutions/facilities/marine\\_performance/towing\\_tank.html](https://www.nrc-cnrc.gc.ca/eng/solutions/facilities/marine_performance/towing_tank.html)
6. Schuster, S., "Beitrag zur Frage der Kanalkorrektur bei Modellversuchen", *Schiffstechnik*, Bd. 3, 1955/56, pp. 93-96
7. Tamura, K., "Study on the Blockage Correction", *Nihon-zosen-gakkai-ronbunshu*, Vol. 131, 1972, pp.17-28
8. Drela, M., Youngren, H., XFOIL Computer Program, [Online]. Available: <http://web.mit.edu/drela/Public/web/xfoil/>
9. Roach, P.E, Turner, J.T., "Secondary loss generation by gas turbine support struts", *Int. J. Heat & Fluid Flow*, Vol. 6, No. 2, 1985, pp. 79-88
10. Rawlings, J.O., Pantula, S.G., Dickey, D.A., *Applied Regression Analysis*, New York, NJ: Springer, 1998
11. <http://www.remmlinger.com/UliTank.html>